

Improper Signaling in Two-Path Relay Channels

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Abstract—Inter-relay interference (IRI) challenges the operation of two-path relaying systems. Furthermore, the unavailability of the channel state information (CSI) at the source and the limited detection capabilities at the relays prevent neither eliminating the interference nor adopting joint detection at the relays nodes. Improper signaling is a powerful signaling scheme that has the capability to reduce the interference impact at the receiver side and improves the achievable rate performance. Therefore, improper signaling is adopted at both relays, which have access to the global CSI. Then, improper signal characteristics are designed to maximize the total end-to-end achievable rate at the relays. To this end, both the power and the circularity coefficient, a measure of the impropriety degree of the signal, are optimized at the relays. Although the optimization problem is not convex, optimal power allocation for both relays for a fixed circularity coefficient is obtained. Moreover, the circularity coefficient is tuned to maximize the rate for a given power allocation. Finally, a joint solution of the optimization problem is proposed using a coordinate descent method based on alternate optimization. The simulation results show that employing improper signaling improves the achievable rate at medium and high IRI.

I. INTRODUCTION

Next generation wireless communication adopts technologies that extends the network coverage and improve the data rate. One of the candidate technologies is full-duplex relaying that targets to double the spectral efficiency. On the other hand, cooperative communication is an interesting technology to improve the data rate and extend the communication range. Full-duplex relaying is employed to extend the network coverage while improving the link quality. Despite of the promising performance that full-duplex can achieve, replacing all half-duplex nodes by full-duplex ones is not possible to be done immediately. During the roll-out phase, half-duplex nodes are used to support full-duplex services. Two path relaying, which is also known as, *alternate relaying*, is a distributed realization of full-duplex relaying. Full-duplex relaying suffers from self-interference, whereas the two-path relaying suffers from inter-relay interference (IRI). Therefore, different interference mitigation techniques need to be adopted to relief the effect of the interference [1].

Improper signaling is used to mitigate the interference impact on communication systems. It is an asymmetric Gaussian signaling scheme that assumes unequal power of the real and imaginary components and/or dependent real and imaginary components. It is used in underlay cognitive radio [2]–[6], overlay cognitive radio [7], full-duplex relaying [8],

Z-interference channel [9], [10] and asymmetric hardware distortions [11]. Recently, we considered the two-path relaying network and showed that improper signaling can be advantageous over proper signaling to mitigate the IRI [12], [13]. Specifically, in [12], improper signaling is adopted in two-path relaying system, where only the same circularity coefficient, a measure of the degree of impropriety of the signal, for both relays is optimized to mitigate the interference while the relays use their maximum power. On the other hand, in [13], we considered the same problem but with different circularity coefficients at the relays. Moreover, we considered asymmetric time allocation for the two transmission phases while the relays use their maximum power.

In this paper, we take the problem in [12] further and optimize both the relay power and circularity coefficient, which measures the degree of impropriety of the transmit signal, to maximize the end-to-end achievable rate of the two-path relaying system. First, we consider proper signaling and introduce optimal relays power allocation for the system. In the case of using improper signaling, we allocate the relays power with a fixed circularity coefficient. Moreover, we tune the circularity coefficient while fixing the transmit power. Then, we jointly optimize the relays power and circularity coefficient via a coordinate descend based method by iterating between the optimal solutions of the individual problems till a convergence obtained. Finally, we investigate through numerical results the merits that can be reaped if the relays use improper signals using different strategies.

II. SYSTEM MODEL

We consider here an alternate two-path relaying network consisting of one source node, S , two half-duplex relay nodes, R_1 and R_2 , and one destination node, D , as shown in Fig. 1. We adopt decode-and-forward protocol at both relays. Moreover, the relays transmit and receive in turn, i.e., in one time slot one relay receives and the other relay transmits, and in the next time slot the other way around. Let h_i and g_i , $i \in \{1, 2\}$, denote the channel between S and R_i and the channel between R_i and D , respectively. We assume channel reciprocity for the inter-relay channel which is denoted by f . Moreover, let us assume that the source transmit power is p_s , the relay transmit power is p_r , and the noise variance at each receiving node is σ_n^2 . The transmit powers are limited to a power budget of p_{\max} . First, we give the following definitions of improper random variables (RV).

Definition 1. [14] The complementary (pseudo-) variance of a zero mean complex random variable x is defined as $\tilde{\sigma}_x^2 = E\{x^2\}$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. If $\tilde{\sigma}_x^2 = 0$, then x is called proper signal, otherwise it is called improper.

Definition 2. [2] The circularity coefficient of the signal x is a measure of its impropriety degree and is defined as $C_x = |\tilde{\sigma}_x^2|/\sigma_x^2$, where $\sigma_x^2 = E\{|x|^2\}$ is the conventional variance and $|\cdot|$ is the absolute value operation. The circularity coefficient satisfies $0 \leq C_x \leq 1$. In particular, $C_x = 0$ and $C_x = 1$ correspond to proper and maximally improper signals, respectively.

We assume that no channel state information is available at S which necessitates the use of proper signaling at S and also makes dirty paper coding of no benefit to fully cancel the IRI. Also, we assume that no direct link is available between S and D . For simplicity and tractability, we consider a yet illustrative scenario by assuming equal power and same circularity coefficient for the relays which may not be optimal. However, as it will be shown in the simulation results, though these sub-optimal assumptions, improper signals show a significantly better performance than proper signaling. Furthermore, we expect even better performance if we increase the degrees of freedom by letting different power and circularity coefficient at the relays. Also, we assume the receivers use the simple practical decoding techniques by treating the interference as a Gaussian noise.

During time slot k , the signal received at R_i with $i = 2 - \text{mod}(k, 2)$ is given by¹

$$y_i[k] = \sqrt{p_s}h_i[k]s[k] + \sqrt{p_r}f[k]x_j[k] + n_i[k], \quad (1)$$

where $s[k]$ is the transmit proper signal by S in time slot k and $n_i[k]$ is the additive noise at R_i with variance σ_n^2 . $x_j[k]$ is the improper signal, with circularity coefficient C_x , transmitted by R_j with $j = 1 + \text{mod}(k, 2)$. The received signal at D from R_i in time slot $k + 1$ is given by

$$y_D[k + 1] = \sqrt{p_r}g_i[k + 1]x_i[k + 1] + n[k + 1], \quad (2)$$

where $n[k + 1]$ is the additive noise at D . In the following, we assume the channels to be quasi-static block flat fading channels and therefore we drop the time index k for notational convenience. The additive noise at the receivers is modeled as a white, zero-mean, circularly symmetric, complex Gaussian with variance σ_n^2 .

The alternating two-path relaying system mimics a full-duplex system by transferring the data through two Z-interference channels, where two transmitters (S and R_i) are sending messages each intended for one of the two receivers (R_j and D) as shown in Fig. 1. Hence, as a result of using improper signals at R_j and proper signals at S while treating the interference as a Gaussian noise, the achievable rate of the

first hop of the i th path ($S - R_i$) can be expressed after some simplification steps as [15]

$$\mathcal{R}_{i,1}(p_r, C_x) = \log_2 \left(1 + \frac{p_s|h_i|^2}{p_r|f|^2 + \sigma_n^2} \right) + \frac{1}{2} \log_2 \left(\frac{1 - C_{y_i}^2}{1 - C_{\mathcal{I}_i}^2} \right), \quad (3)$$

where C_{y_i} and $C_{\mathcal{I}_i}$ are the circularity coefficients of the received and interference-plus-noise signals at R_i , respectively, which can be calculated as

$$C_{y_i} = \frac{p_r|f|^2 C_x}{p_s|h_i|^2 + p_r|f|^2 + \sigma_n^2}, \quad C_{\mathcal{I}_i} = \frac{p_r|f|^2 C_x}{p_r|f|^2 + \sigma_n^2}. \quad (4)$$

Hence, (3) can be simplified to

$$\mathcal{R}_{i,1}(p_r, C_x) = \frac{1}{2} \times \log_2 \left(1 + \frac{2p_s|h_i|^2 (p_r|f|^2 + \sigma_n^2) + p_s^2|h_i|^4}{(1 - C_x^2)p_r^2|f|^4 + 2p_r|f|^2\sigma_n^2 + \sigma_n^4} \right). \quad (5)$$

Similarly, the achievable rate of the second hop of the i th path can be obtained from (2) as

$$\mathcal{R}_{i,2}(p_r, C_x) = \log_2 \left(1 + \frac{p_r|g_i|^2}{\sigma_n^2} \right) + \frac{1}{2} \log_2 \left(\frac{1 - C_{y_D}^2}{1 - C_{\mathcal{I}_D}^2} \right), \quad (6)$$

where C_{y_D} and $C_{\mathcal{I}_D}$ are the circularity coefficients of the received and interference-plus-noise signals at D , respectively, which can be computed as

$$C_{y_D} = \frac{p_r|g_i|^2 C_x}{p_r|g_i|^2 + \sigma_n^2}, \quad C_{\mathcal{I}_D} = 0. \quad (7)$$

Then, (6) reduces to

$$\mathcal{R}_{i,2}(p_r, C_x) = \frac{1}{2} \log_2 \left(1 + \frac{2p_r|g_i|^2}{\sigma_n^2} + \frac{p_r^2|g_i|^4 (1 - C_x^2)}{\sigma_n^4} \right). \quad (8)$$

Hence, the end-to-end achievable rate of the i th path can be calculated from

$$\mathcal{R}_i(p_r, C_x) = \min \left\{ \mathcal{R}_{i,1}(p_r, C_x), \mathcal{R}_{i,2}(p_r, C_x) \right\}. \quad (9)$$

Accordingly, the overall end-to-end achievable rate of the two-path relaying system, for sufficiently large number of time slots², is expressed as the arithmetic mean of $\mathcal{R}_i(p_r, C_x)$

$$\mathcal{R}_T(p_r, C_x) = \frac{1}{2} \sum_{i=1}^2 \mathcal{R}_i(p_r, C_x). \quad (10)$$

Remark 1. One can notice that if $C_x = 0$ in (10), we obtain the conventional expression for the total achievable rate of the two-path relaying system under the use of proper signals as

$$\mathcal{R}_T(p_r, 0) = \frac{1}{2} \sum_{i=1}^2 \mathcal{R}_i(p_r, 0) = \frac{1}{2} \times \sum_{i=1}^2 \min \left\{ \log_2 \left(1 + \frac{p_s|h_i|^2}{\sigma_n^2 + p_r|f|^2} \right), \log_2 \left(1 + \frac{p_r|g_i|^2}{\sigma_n^2} \right) \right\}. \quad (11)$$

²One slot is missed at the start of the transmission without delivering information from S to D .

¹For the rest of the paper, we let $i, j \in \{1, 2\}$, $i \neq j$

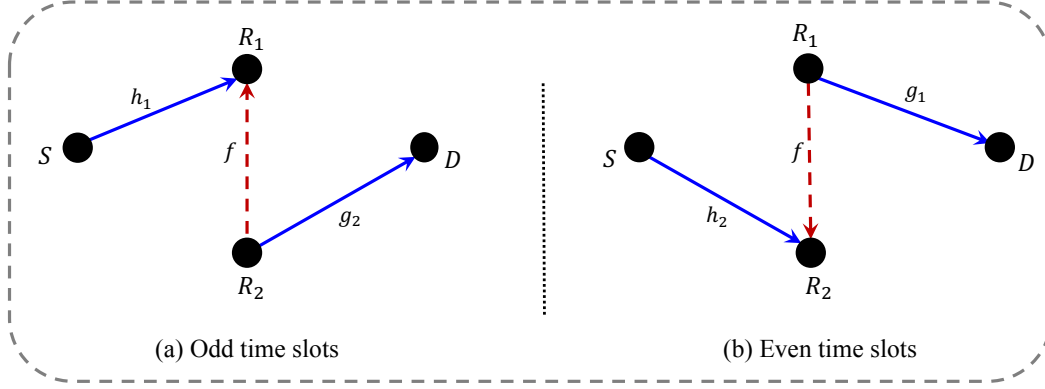


Fig. 1: Two-Path Relay Channel. The blue solid lines represent the signal links and the red dashed lines represent the IRI.

III. IMPROPER GAUSSIAN SIGNALING DESIGN FOR TWO-PATH RELAYING SYSTEMS

In this section, we aim at optimizing the relays signal parameters represented in the relay's transmit power p_r and the circularity coefficient \mathcal{C}_x in order to maximize the instantaneous end-to-end achievable rate of the system. First, the intuition behind the benefit of using improper signals at the relays is that it provides an additional degree of freedom that can be optimized in order to alleviate the effect of the IRI on the relays or, in the worst case, kept at the same performance as proper signaling, i.e., $\mathcal{C}_x = 0$. Moreover, improper signaling has the ability to control the interference signal dimension, and it is one form of interference alignment [10], [16]. Furthermore, when using proper signals, R_i can improve the rate of the second hop of the i th by boosting its transmit power. However, this will deteriorate the rate of the first hop of the j th path and here improper signaling attains its benefit. By increasing the asymmetry of the relay's transmit signal, by boosting the circularity coefficient, the relay can increase its power and has a less adverse effect on the other one.

Now, in order to reap the benefits of improper signaling, we design the power and circularity coefficient of the relays. For this purpose, we formulate the following optimization problem

$$\begin{aligned} \mathbf{P1} : \max_{p_r, \mathcal{C}_x} \quad & \mathcal{R}_T(p_r, \mathcal{C}_x) \\ \text{s.t.} \quad & 0 < p_r \leq p_{\max}, \\ & 0 \leq \mathcal{C}_x \leq 1. \end{aligned} \quad (12)$$

Solving **P1** optimally is difficult as it is a non-convex optimization problem. Here, we propose a coordinate-descent (CD) based method in which we consider two problems, optimizing the relays transmit power for a fixed circularity coefficient and optimizing the circularity coefficient for a fixed transmit power. Finally, we perform alternate optimization of the optimal solutions of the two problems till we get convergence.

Remark 2. [17] The CD method is popular for its efficiency, simplicity and scalability. Moreover, it is guaranteed to converge to a local solution if the global optimal solution is attained for each of the sub-problems. However, it does

not necessarily converge to the global optimal solution as the objective function is non-convex.

Following Remark 2, we will show the optimal solutions of the two sub-problems. First, for notational convenience, we give the following definitions.

Definition 3. Let π denote the permutation of $\{1, 2\}$ that sets the points $z_i \in \mathbb{R}_{++}$ in an increasing order such that $z_{\pi_1} \leq z_{\pi_2}$. Also, let $\mathcal{F}_{i,j}(p_r, \mathcal{C}_x) = \mathcal{R}_{i,1}(p_r, \mathcal{C}_x) + \mathcal{R}_{j,2}(p_r, \mathcal{C}_x)$ and $k_i(p_r, \mathcal{C}_x) = \arg \min_{a \in \{1,2\}} \mathcal{R}_{i,a}(p_r, \mathcal{C}_x)$.

Sub-problem 1) Relays Transmit Power Optimization Problem

In this part, we optimize the relays transmit power for a fixed circularity coefficient \mathcal{C}_x^o . The corresponding optimization problem is given by

$$\begin{aligned} \mathbf{P2}(\mathcal{C}_x^o) : \max_{p_r} \quad & \mathcal{R}_T(p_r, \mathcal{C}_x^o) \\ \text{s.t.} \quad & 0 < p_r \leq p_{\max}. \end{aligned} \quad (13)$$

It can be verified that **P2** is a non-convex optimization problem which makes it hard, in general, to find its optimal solution. Also, due to the coupling between the achievable rates of the two paths in terms of p_r , maximizing the rates of each individual path with respect to p_r and taking the arithmetic mean is not optimal. However, thanks to some special monotonicity properties of the objective function, we show that the optimal solution of **P2** lies either at the intersection between $\mathcal{R}_{i,1}(p_r, \mathcal{C}_x^o)$ and $\mathcal{R}_{i,2}(p_r, \mathcal{C}_x^o)$, if exists or one of the stationary points of the $\mathcal{F}_{i,j}(p_r, \mathcal{C}_x^o)$ with respect to p_r , if exists or the power budget p_{\max} . Next, we will compute the intersection and stationary points.

Proposition 1. There exists at most one intersection point, p_i , between $\mathcal{R}_{i,1}(p_r, \mathcal{C}_x^o)$ and $\mathcal{R}_{i,2}(p_r, \mathcal{C}_x^o)$ over the feasible interval $0 < p_r \leq p_{\max}$. Moreover, this intersection point can be obtained by solving the quartic equation³ in (14).

Proof. By equating $\mathcal{R}_{i,1}(p_r, \mathcal{C}_x^o)$ and $\mathcal{R}_{i,2}(p_r, \mathcal{C}_x^o)$, we obtain (14). Then, by arranging the coefficients of the quartic equa-

³The quartic equation can be solved by Ferrari's method [18]. However, since the roots derived from this quartic equation are extremely complex and lengthy, we omit them due to the space limitations.

$$\begin{aligned} & \frac{|g_i|^4 |f|^4 (1 - C_x^2)^2}{\sigma_n^4} p_i^4 + \frac{2 |g_i|^2 |f|^2 (|g_i|^2 + |f|^2) (1 - C_x^2)}{\sigma_n^2} p_i^3 + |g_i|^2 (4 |f|^2 + |g_i|^2 (1 - C_x^2)) p_i^2 \\ & + 2 (\sigma_n^2 |g_i|^2 - p_s |h_i|^2 |f|^2) p_i - (p_s^2 |h_i|^4 + 2 p_s |h_i|^2 \sigma_n^2) = 0. \end{aligned} \quad (14)$$

tion in a descending order, the signs of these coefficients, according to the sign of the linear term is either $\{+, +, +, +, -\}$ or $\{+, +, +, -, -\}$. In both cases, there is only one change of signs. For our real quartic polynomial, this determines the number of positive roots to be exactly one root over \mathbb{R}_{++} by using Descartes rule of signs [19]. Hence, there exists at most one intersection point over the feasible interval. \square

Remark 3. For the case of using proper signals at the relays i.e., $C_x = 0$, the quartic equation reduces to the following quadratic equation

$$p_i^2 + \frac{\sigma_n^2}{|f|^2} p_i - \frac{p_s |h_i|^2 \sigma_n^2}{|g_i|^2 |f|^2} = 0, \quad (15)$$

which can be solved to obtain the intersection point as

$$p_i = \frac{1}{2|f|} \left(\sqrt{\frac{\sigma_n^4}{|f|^2} + \frac{4p_s |h_i|^2 \sigma_n^2}{|g_i|^2}} - \frac{\sigma_n^2}{|f|} \right). \quad (16)$$

Now, We can divide \mathbb{R}_{++} into three intervals where $\{0, p_{\pi_1}, p_{\pi_2}, \infty\}$ are the boundaries for these intervals. From (10), $\mathcal{R}_T(p_r, C_x)$ can be reformulated in each interval as

$$\mathcal{R}_T(p_r, C_x) = \frac{1}{2} \times \begin{cases} \sum_{i=1}^2 \mathcal{R}_{i,2}(p_r, C_x), & \text{if } 0 < p_r \leq p_{\pi_1} \\ \mathcal{R}_{\pi_1,1}(p_r, C_x) + \mathcal{R}_{\pi_2,2}(p_r, C_x), & \text{if } p_{\pi_1} < p_r \leq p_{\pi_2} \\ \sum_{i=1}^2 \mathcal{R}_{i,1}(p_r, C_x), & \text{if } p_{\pi_2} < p_r < \infty \end{cases}. \quad (17)$$

There are at maximum five stationary points $p_{st_i}^{(n)} \in \mathbb{C}$, $n \in \{1, 2, 3, 4, 5\}$ of $\mathcal{F}_{i,j}(p_r, C_x)$ which can be calculated by finding the roots of the derivative of $\mathcal{F}_{i,j}(p_r, C_x)$ with respect to p_r over the interval $0 < p_r \leq p_{\max}$. The resulting equation is a quintic equation⁴, which is very lengthy and we omit it due to space limitation.

Remark 4. When using proper signals at the relays, the quintic equation reduces to the quadratic equation

$$\frac{|g_j|^2}{\sigma_n^2} (\sigma_n^2 + p_{st_i} |f|^2)^2 = p_s |h_i|^2 (|f|^2 - |g_j|^2), \quad (18)$$

which can be solved to get only one possible stationary point as

$$p_{st_i} = \sqrt{\frac{\sigma_n^2 p_s |h_i|^2}{|g_j|^2 |f|^4} (|f|^2 - |g_j|^2)} - \frac{\sigma_n^2}{|f|^2}, \quad (19)$$

⁴The feasible roots of the quintic equation can be obtained numerically.

in which it can be easily shown that $p_{st_i} \in \mathbb{R}_{++}$ if and only if

$$|f|^2 - |g_j|^2 > \frac{\sigma_n^2 |g_j|^2}{p_s |h_i|^2}. \quad (20)$$

Before introducing the optimal solution of **P2**, let us give the following definition

Definition 4. Let the set of feasible transmit powers $\mathcal{P}_{\text{int}} = \{p_i \mid 0 < p_i \leq p_{\max}\}$. Also, the set of feasible stationary points $\mathcal{P}_{\text{st}} = \{p_{st_i}^{(n)} \mid p_{\pi_1} < p_{st_i}^{(n)} \leq p_{\pi_2} \text{ \& } p_{st_i}^{(n)} \leq p_{\max}\}$.

From Definition 4, \mathcal{P}_{int} and \mathcal{P}_{st} can be empty sets. Based on the aforementioned analysis, the optimal solution of **P2** can be found from the following theorem.

Theorem 1. In a two-path relaying system, where the two relays transmit improper signals and by treating interference as a Gaussian noise, the optimal power allocation, at a fixed circularity coefficient, that maximizes the total achievable rate constrained by a power budget p_{\max} can be obtained as

$$p_r^* = \arg \max_{p \in \mathcal{P}_T} \mathcal{R}_T(p, C_x), \quad (21)$$

where $\mathcal{P}_T = \mathcal{P}_{\text{int}} \cup \mathcal{P}_{\text{st}} \cup p_{\max}$.

Proof. From the definition of the total rate function in (17), it can be readily verified that the function in the first interval, i.e., $0 < p_r \leq p_{\pi_1}$, is monotonically increasing in p_r , thus the optimal solution of **P2** in this interval is p_{π_1} . Moreover, the function in (17) in the last interval, i.e., $p_{\pi_2} < p_r < \infty$, is monotonically decreasing in p_r and hence the optimal solution in this interval is p_{π_2} . If the maximum of R_T is in the middle interval, it must occur at a stationary point. Finally, we limit these points by the power budget and this concludes the proof. \square

Sub-problem 2) Circularity Coefficient Optimization Problem

Now, we optimize the propriety of the relays transmit signal, measured by the circularity coefficient, assuming a fixed transmit power p_r^o . To this end, we formulate the following optimization problem.

$$\begin{aligned} \mathbf{P3}(p_r^o) : & \max_{C_x} \mathcal{R}_T(p_r^o, C_x) \\ \text{s.t.} & \quad 0 \leq C_x \leq 1. \end{aligned} \quad (22)$$

This problem has been addressed in our work [12] and the optimal solution is given in the following theorem.

Theorem 2. [12] In a two-path relaying system, where the two relays transmit improper signals and by treating interference as a Gaussian noise, the optimal circularity coefficient,

at a fixed relay transmit power, that maximizes the total achievable rate can be obtained as

Case 1: no intersection points

$$\mathcal{C}_x^* = \begin{cases} 0, & \text{if } k_1(\mathcal{C}) = k_2(\mathcal{C}) = 2, 0 \leq \mathcal{C} \leq 1 \\ 1, & \text{if } k_1(\mathcal{C}) = k_2(\mathcal{C}) = 1, 0 \leq \mathcal{C} \leq 1 \\ \arg \max_{\mathcal{C}_x \in \{0, \mathcal{C}_{\text{st}_i}, 1\}} \mathcal{F}_{i,j}(p_r^o, \mathcal{C}_x), & \text{if } k_1(\mathcal{C}) = i, k_2(\mathcal{C}) = j, 0 \leq \mathcal{C} \leq 1 \end{cases} \quad (23)$$

Case 2: one intersection point, \mathcal{C}_i

$$\mathcal{C}_x^* = \begin{cases} \arg \max_{\mathcal{C}_x \in \{\mathcal{C}_i, \mathcal{C}_{\text{st}_j}, 1\}} \mathcal{F}_{j,i}(p_r^o, \mathcal{C}_x), & \text{if } k_j(\mathcal{C}) = 1, 0 \leq \mathcal{C} \leq 1 \\ \arg \max_{\mathcal{C}_x \in \{0, \mathcal{C}_{\text{st}_i}, \mathcal{C}_i\}} \mathcal{F}_{i,j}(p_r^o, \mathcal{C}_x), & \text{if } k_j(\mathcal{C}) = 2, 0 \leq \mathcal{C} \leq 1 \end{cases} \quad (24)$$

Case 3: two intersection points, $(\mathcal{C}_{\pi_1}, \mathcal{C}_{\pi_2})$

$$\mathcal{C}_x^* = \arg \max_{\mathcal{C}_x \in \{\mathcal{C}_{\pi_1}, \mathcal{C}_{\text{st}_{\pi_2}}, \mathcal{C}_{\pi_2}\}} \mathcal{F}_{\pi_2, \pi_1}(p_r^o, \mathcal{C}_x). \quad (25)$$

where \mathcal{C}_i and $\mathcal{C}_{\text{st}_i}$ are the intersection between $\mathcal{R}_{i,1}(p_r^o, \mathcal{C}_x)$ and $\mathcal{R}_{i,2}(p_r^o, \mathcal{C}_x)$ and the stationary point for $\mathcal{F}_{i,j}(p_r^o, \mathcal{C}_x)$ with respect to \mathcal{C}_x , over the feasible interval $0 < \mathcal{C}_x \leq 1$, respectively⁵.

Proof. For more clarity of the proof of this theorem in [12] and for the reader's convenience and also to gain more insights of the solution of **P3**, we additionally graph the possible configurations of the rate functions $\mathcal{R}_{i,j}(p_r^o, \mathcal{C}_x)$ in (5) and (8) over $0 \leq \mathcal{C}_x \leq 1$ in Appendix A, which is available online at [20] due to space limitations. \square

Coordinate Descent: Joint Optimization Problem

Here, we aim at optimizing jointly the relays power and circularity coefficient in order to maximize the total rate of the two-path relaying system via CD, in which we implement alternate optimization of p_r and \mathcal{C}_x . In this method, we optimize the transmit power for a fixed circularity coefficient. Then, we use the optimal power in the previous step to optimize for the circularity coefficient and iterate between the optimal solutions till a stopping criterion is satisfied. For this purpose, we develop Algorithm I to obtain the optimization parameters of **P1**.

Algorithm I: Joint Alternate Optimization of the power and circularity coefficient based on the CD method.

- 1: **Input** $h_i, g_i, f, \sigma_n^2, p_{\max}, \epsilon_{\max}, 0 < p_r^o \leq p_{\max}, 0 \leq \mathcal{C}_x \leq 1$.
 - 2: **Initialize** $p_r \leftarrow p_r^o, \mathcal{C}_x \leftarrow \mathcal{C}_x^o$ and $\epsilon \leftarrow \infty$
 - 3: **while** $\epsilon > \epsilon_{\max}$ **do**
 - 4: **Compute** \hat{p}_r from **P2** (\mathcal{C}_x) using Theorem 1
 - 5: **Compute** $\hat{\mathcal{C}}_x$ from **P3** (\hat{p}_r) using Theorem 2
 - 6: **Set** $\epsilon = \max\{|\hat{\mathcal{C}}_x - \mathcal{C}_x|, |\hat{p}_r - p_r|\}$ %max error
 - 7: **Update** $p_r \leftarrow \hat{p}_r$
 - 8: **Update** $\mathcal{C}_x \leftarrow \hat{\mathcal{C}}_x$
 - 9: **end while**
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⁵For more about the existence and uniqueness of \mathcal{C}_i and $\mathcal{C}_{\text{st}_i}$, please refer to [12].

IV. NUMERICAL RESULTS

In this section, we numerically evaluate the average end-to-end rate of the proposed two-path relaying system using improper signaling. Throughout the following simulation scenarios, we compare between proper and improper signaling. For proper based system, we include two scenarios: maximum power allocation (MPA) and optimal power allocation (OPA). On the other hand for improper based system, we include three scenarios: MPA for maximally improper relay signal, i.e., $\mathcal{C}_x = 1$, optimized CD based method using an initial point for the power as $p_r^o = p_{\max}$ and two different initial starting points for the circularity coefficient; $\mathcal{C}_x^0 = 0$ and $\mathcal{C}_x^0 = 1$ and the joint optimal allocation of p_r and \mathcal{C}_x using a fine exhaustive grid search (GS) as a benchmark for the alternate optimization. The average channel signal-to-noise ratios (SNRs) are defined as $\gamma_{h_i} = \sigma_{h_i}^2 / \sigma_n^2$, $\gamma_{g_i} = \sigma_{g_i}^2 / \sigma_n^2$ and $\gamma_f = \sigma_f^2 / \sigma_n^2$. The results are averaged over 10000 channel realizations and $\epsilon_{\max} = 0.0001$.

As for the simulation setup, we assume symmetric relays links with zero-mean complex Gaussian distribution and $\gamma_{h_i} = 10$ dB, $\gamma_{g_i} = 15$ dB, $\gamma_f = 20$ dB, unless otherwise specified.

Firstly, to explore the impact of improper signaling on two-path relaying systems, we study the average rate performance versus γ_f as can be seen in Fig 2. It is clear that, proper and improper based systems suffer from a rate degradation as the interference link increases which worsen the performance of S-R links and thus limits the end-to-end rate. For the proper based system, we observe that optimizing the relay power reduces the IRI impact on the relays and improve the rate. As for improper signaling, optimizing \mathcal{C}_x with maximum power can significantly boost the rate at mid and high interference levels. At low interference levels, improper-MPA achieves better performance than proper-MPA, however it can not compete with proper-OPA as the interference is not dominant in such situation and thus proper signaling becomes preferable. The same observation is observed for other improper based systems when compared with proper-MPA.

As for CD joint optimization solution, the proper choice of initial points in CD plays an important role in the overall performance compared with the GS solution as can be observed in Fig. 2. As a result, starting the CD with $\mathcal{C}_x^0 = 1$ can converge to the GS solution while $\mathcal{C}_x^0 = 0$ improves the rate performance but it does not converge to the optimal performance. This observation can be justified as the solution at high interference levels reduces to maximally improper, i.e., $\mathcal{C}_x^* = 1$ as can be seen from the improper-MPA system.

Secondly, we study the average end-to-end rate performance of the aforementioned system versus γ_{h_i} as can be shown in Fig. 3. At very low γ_{h_i} values, the first hops become a bottleneck and degrade the end-to-end average rate for both proper and improper based systems. As γ_{h_i} increases, improper systems use more transmit powers and alleviate the IRI through the increase of the signal propriety by boosting the circularity coefficient while proper-MPA systems use relatively less power. This improvement gap remains until the value of γ_{h_i} becomes relatively large with respect to γ_f , and hence the proper based system starts to enhance its performance

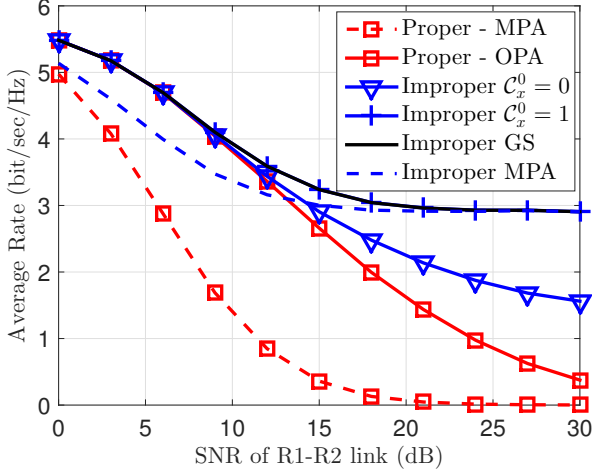


Fig. 2: The average achievable end-to-end rate for proper and improper signaling with different methods versus γ_f .

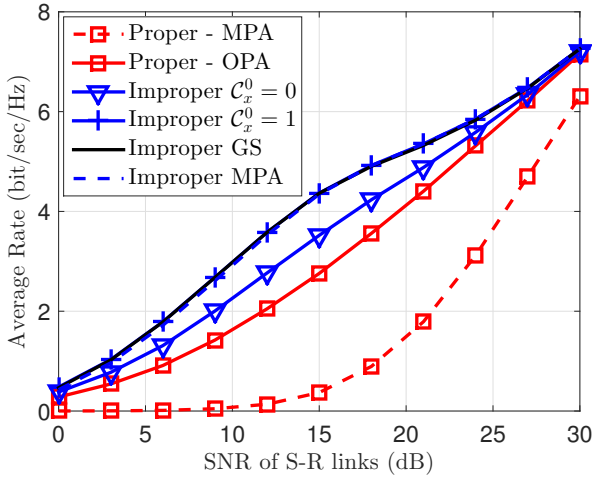


Fig. 3: The average achievable end-to-end rate for proper and improper signaling with different techniques versus γ_{h_i} .

by increasing its transmit power. At high γ_{h_i} , both systems tend to utilize the power budget and the improper solution reduces to proper. From this investigation, we can state that improper signaling is preferred when the first hops become a bottleneck. As expected from the previous simulation scenario at $\gamma_f = 20$ dB, improper-MPA achieves a close performance to the improper-GS.

V. CONCLUSION

In this paper, we propose to use improper signaling in order to mitigate the inter-relay interference (IRI) in two-path relaying systems. First, we formulate an optimization problem to tune the relays transmit power and the circularity coefficient, a measure of the degree of asymmetry of the signal, to maximize the total end-to-end achievable rate of the two-path relaying system considering a power budget. We first introduce the optimal allocation of the relays power at a fixed circularity coefficient to maximize the achievable rate, then

we optimize the circularity coefficient at a fixed relays power. After that we numerically optimize the relays power and circularity coefficient jointly through a coordinate descent based method. The numerical results show a significant improvement of the total rate when the relays transmit improper signals, specifically, at mid and high IRI values. More generally, the merits of using improper signaling become significant when the first hop is the bottleneck of the system due to either weak gains or the excess of IRI.

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